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THERMAL INSTABILITY OF THE HELIUM-BURNING SHELL IN MASSIVE STARS

RICHARD STOTHERS AND CHAO-WEN CHIN
Institute for Space Studies, Goddard Space Flight Center, NASA, New York, New York
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ABSTRACT

Nonlinear numerical calculations of stellar evolution at high mass show that thermal instability develops temporarily in the helium-burning shell, shortly after the ignition of shell helium. Manifestation of the instability is an irregular "flickering" of very small amplitude. There appear to be no observable consequences of the phenomenon.
Subject headings: instabilities — interiors, stellar — massive stars — stellar evolution

I. INTRODUCTION

Thermal instability in the helium-burning shell of a star has so far been found only in models of stars with low to intermediate masses. Since radiation pressure is important at higher masses and tends to oppose thermal instability, it has generally been supposed that the helium-burning shell in a massive star is probably thermally stable (e.g., Hōshi 1968). An explicit linearized calculation by Dennis (1971) for a model of a star of 15 $M_\odot$ in the phase of carbon-core contraction has supported this belief.

In a recent paper (Stothers and Chin 1972), we have found that, under certain conditions in which the *hydrogen-burning* shell of a massive star is initially formed near a hydrogen discontinuity, this shell becomes temporarily unstable. Since the conditions under which the helium-burning shell is formed are quite similar, we have decided to reinvestigate the question of thermal instability in the helium-burning shell of massive stars.

II. MODEL STRUCTURES

The present study is based on the same methods and input physics as were used in our two previous studies of the thermal stability of the hydrogen-burning shell (Stothers and Chin 1972, 1973). In addition, we mention the following supplementary data: (1) the nuclear energy-generation rate for the $^{12}C + ^{12}C$ reaction, taken from Arnett and Truran (1969) but multiplied by a factor of 4 to take account roughly of the latest cross-section measurements (Mazarakis and Stephens 1972; Michaud 1972); (2) the omission of neutrino losses; (3) the neglect of a small correction for electron degeneracy in the equation of state; and (4) the neglect of convective overshooting and of semiconvection at the boundary of the helium-burning convective core.

Evolutionary sequences for stars of 15 and 30 $M_\odot$ were computed from the last stages of core helium burning ($Y_c = 0.01$) to the onset of core carbon burning ($^{12}C + ^{12}C$ reaction), with the use of time steps small compared with the thermonuclear e-folding time of the helium-burning shell (eq. [4] of Stothers and Chin 1972). Initial models for the evolutionary sequences were taken from our most recent paper (Stothers and Chin 1973). Table 1 lists the adopted sequences, in the same notation that we used previously. Notice that the most recent rate of nuclear energy generation for the 3α process has been adopted (Austin, Trentelman, and Kashy 1971). A subscript H or He refers to the hydrogen or helium shell, respectively, and q stands for stellar mass fraction while Δq refers to the shell mass thickness. Sequence 15-A has been selected

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TABLE 1

CHARACTERISTICS OF THE EVOLUTIONARY SEQUENCES OF MODELS FOR STARS OF 15 AND 30 $M_\odot$ DURING THE PHASE OF CARBON-CORE CONTRACTION

Sequence	15-A	15-F	15-O	30-B	30-C
X_e	0.739	0.739	0.602	0.739	0.602
Z_e	0.021	0.044	0.021	0.044	0.021
θ_α^2	0.0	0.1	0.1	0.1	0.1
$\epsilon_{3\alpha}$	New	New	New	New	New
X_C (center)	0.979	0.155	0.155	0.132	0.100
q_H	0.27	0.25	0.37	0.34	0.46
q_{He}	0.13	0.12	0.21	0.25	0.36
$(\Delta q)_{He}$	0.02	0.02	0.03	0.02	0.03
Smallest $\langle dY/dq \rangle_{He}$	30	50	40	40	30
$1 - \beta_{He}$	0.3	0.3	0.4	0.5	0.6

as our "standard" sequence. Sequences 15-F and 15-O have been chosen mainly to test the effect of different locations of the helium shell. The same reasoning applies to the choice of sequences 30-B and 30-C, apart from the obvious interest in studying a higher stellar mass. The total range of carbon-core masses covered in the present work is 2–11 $M_\odot$.

A brief description of the overall evolutionary changes that occur inside massive stars after the end of core helium burning will be useful. As helium becomes exhausted near the center, the core (which is nondegenerate) contracts and heats up rapidly, and the surrounding helium-rich layers ignite. A helium-burning shell is then formed. When the central helium abundance drops identically to zero, the shrinking convective region near the center also disappears, only to reappear and to grow in mass fraction when central carbon is later ignited. Throughout all, or at least the last, stages of carbon-core contraction, the star is observationally a red supergiant and has a steadily deepening convective envelope.

In all our model sequences, the convective envelope penetrates down close to the hydrogen shell, but never past it, even when it ceases to burn shortly after the ignition of shell helium. This result is in contrast to the deeper convective envelopes found in our earlier semianalytical models (Stothers and Chin 1969). The difference seems to be due to the larger core masses of our earlier models (traceable to the adoption of only electron scattering for the interior opacity) and, possibly, to the crudity of our previously used envelope boundary conditions. Another reason may be the multiplicity of static solutions that we found for models with heavy-ion cores (Stothers and Chin 1969). In any event, we confirm that the (uncertain) details of the earlier evolutionary history of the layers overlying the hydrogen shell are obliterated by the deep convective envelope during the phase of carbon-core contraction.

III. NONLINEAR STUDY OF THERMAL PULSES

Thermal instability within the helium-burning shell develops temporarily at some stage or other in all of our sequences tested. The reason for this has been essentially predicted in our earlier paper (Stothers and Chin 1972). When the helium shell first forms, it is stable because its temperature and luminosity are low, and the sharp chemical discontinuity ensures that the shell is very narrow. After a while, helium burning becomes more vigorous, produces a brighter luminosity, and rounds out the discontinuity (making the shell broader), and so the tendency to thermal instability is greater. Eventually, as the shell temperature continues to mount, the helium gradient through the shell steepens, and the shell once again becomes narrow. The helium shell profile at several stages of evolution is shown in figure 1.

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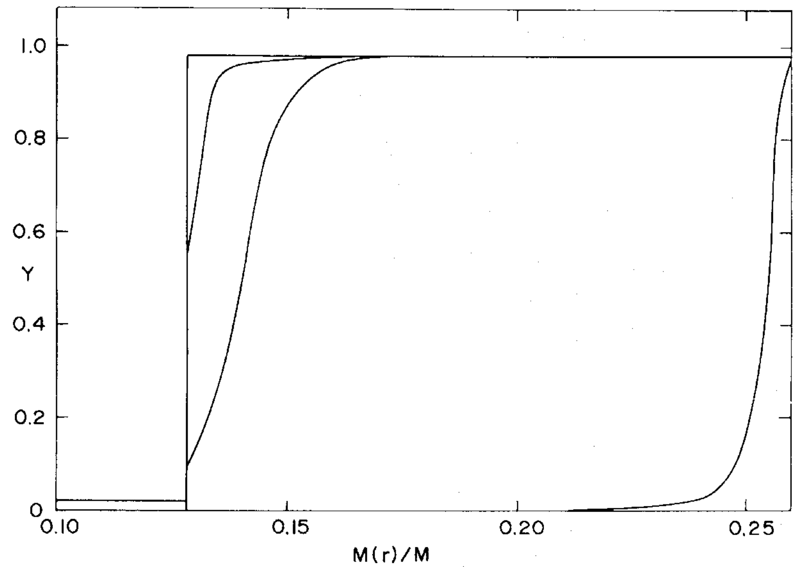

FIG. 1.—Helium profile in the vicinity of the helium-burning shell in a star of 15 $M_\odot$ (sequence 15-A), for four successive stages of evolution: (1) helium-shell ignition; (2) onset of thermal instability in the helium shell; (3) onset of core carbon burning (helium shell is still thermally unstable); and (4) end of core carbon burning (helium shell is again thermally stable).

In terms of an approximate analytic criterion for thermal instability derived by Hōshi (1968) and used earlier (eq. [2] of Stothers and Chin 1972), the magnitude of the true mass of the shell relative to a "critical" mass, $\Delta M/(\Delta M)_{\rm crit}$ indicates the possible occurrence of thermal instability. It is necessary (though not sufficient) that $\Delta M/(\Delta M)_{\rm crit}$ exceed a value of ~ 0.5. This numerical value seems to be in good agreement with our present models since $\Delta M/(\Delta M)_{\rm crit} \approx 0.5$ when the thermal pulses first appear (before helium vanishes at the stellar center and before the hydrogen shell is extinct, in the case of 15 $M_\odot$) and when they first disappear. About 4×10^4 years after the first pulse in the case of 15 $M_\odot$ or 2×10^4 years in the case of 30 $M_\odot$, $\Delta M/(\Delta M)_{\rm crit}$ reaches maximum values of ~ 2.0 and ~ 0.8, respectively. Depending on the mass and (more critically) on the initial carbon abundance of the core, this maximum value may occur before, during, or after core carbon burning. (Sequences 15-A and 30-C were continued to the end of core carbon burning, and the helium-burning shell became thermally stable well before the end in sequence 15-A but remained thermally unstable even after the end in sequence 30-C.) The thermonuclear *e*-folding time of the helium shell remains roughly constant from the time of the first pulse until the onset of core carbon burning; it has an average value of ~ 200 years for 15 $M_\odot$ and ~ 800 years for 30 $M_\odot$, but it reverses its trend inversely with the trend of $\Delta M/(\Delta M)_{\rm crit}$.

The question now arises as to why Dennis (1971) did not find thermal instability in the helium-burning shell of a 15 $M_\odot$ model taken from one of our earlier semi-analytical sequences (Stothers and Chin 1969). This model was constructed on the assumptions of purely electron-scattering opacity, the old 3α rate given by Reeves (1965), $\theta_\alpha{}^2 = 0$, $q_{\rm He} = 0.24$, and $q_{\rm H} = 0.27$. We have rerun the same sequence with our present computer program and have verified Dennis's result that the helium-burning shell is thermally stable up to core carbon burning. The reason lies in the large value of $q_{\rm He}$, which is not much less than $q_{\rm H}$. This proximity causes the hydrogen shell to remain active, with the consequence that the burning in the helium shell is feebler than it would otherwise be [$\Delta M/(\Delta M)_{\rm crit}$ is smaller]. Eventually, thermal

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pulses in the helium-burning shell do develop, but only after 2 percent of the carbon in the core has been depleted.

To check further on the influence of parameters other than shell position on the thermal stability of the helium shell, we have made three reruns of sequence 15-A, making the following individual changes: (1) an initial rounding of the helium profile in the shell, as might be due to convective overshooting of the receding helium-burning core; (2) the inclusion of the $^{12}C + \alpha$ reaction, with $\theta_\alpha^2 = 1$ (however, this reaction has nearly the same temperature dependence as does the 3α process); and (3) the inclusion of neutrino emission, with the standard cross-sections. None of these changes is found to cause any significant difference from our previous results for sequence 15-A. One last test that we made concerns the thermal stability of the hydrogen-burning shell before it becomes extinct; we found, as expected, that this shell is thermally stable.

The thermal instability of the helium-burning shell discovered in the present investigation differs in two important respects from the instability found in all previous investigations. First, the pulses found in the present work are of extremely low amplitude; generally, the amplitude of the shell peak temperature $\delta \log T_{\text{He}} \leq 0.002$, and there is no detectable fluctuation of the surface luminosity. (The threshold of detectability for $\delta \log T_{\text{He}}$ and $\delta \log L$ is ~ 0.0002 in our work.) Second, the amplitude and frequency of the pulses are often erratic. Thus the phenomenon of thermal instability in the present models can best be described as a kind of irregular "flickering" of the helium shell.

The reasons for the irregularity are unknown, but the smallness of the pulse amplitudes can be explained by the large radiation pressure and large surface area of the burning shell in massive stars (Hōshi 1968; Stothers and Chin 1972). As might be expected, the pulse amplitudes tend to be somewhat larger when the shell mass thickness is large, but, on the other hand, this occurs when the radiation pressure and surface area of the shell are also large (see table 1). Therefore, a delicate balance between competing factors barely allows the pulses to develop at all. In low-mass stars, the helium shell has a mass thickness which is one to two orders of magnitude smaller than that existing in high-mass stars; however, radiation pressure and shell area are also much smaller in the low-mass stars.

The thermal pulses in massive stars are too feeble to generate any significant convective mixing in the vicinity of the helium-burning shell. Therefore, their consequences for the overall evolution are negligible. But because gas pressure in the nondegenerate stellar core is temperature-dependent and because high radiation pressure tends to make pressure perturbations spatially constant, the thermal pulses in the helium-burning shell (as small as they are) create noticeable fluctuations of the central temperature. This is in contrast to the situation in stars of very low mass, where degenerate gas pressure dominates in the core.

A number of similarities between the pulses in massive stars and those in low-mass stars are worth pointing out. A rough mean cycle time of ~ 4000 years seems to apply to the pulses in most of our sequences, even though the deviation of an individual interpulse period occasionally exceeds 50 percent. In comparison, several thousand years is also characteristic of the cycle time of pulses found in models for 0.75 $M_\odot$ (Rose 1967) and 5 $M_\odot$ (Weigert 1966), although a cycle time one to two orders of magnitude longer than this occurs in models for 0.53 $M_\odot$ (Rose 1966), 0.85 $M_\odot$ (Rose and Smith 1970), and 1 $M_\odot$ (Schwarzschild and Härm 1967, 1970; Vila 1970; Sweigart 1971; Härm and Schwarzschild 1972). A thousand years is also characteristic of the cycle time of pulses found in the hydrogen-burning shell of certain models for 15, 30, and 60 $M_\odot$ just after core hydrogen burning (Stothers and Chin 1972). Although no "secondary pulse" trailing the main pulse of each cycle ever occurred in these unstable hydrogen-burning shells, our present models occasionally do show a secondary pulse

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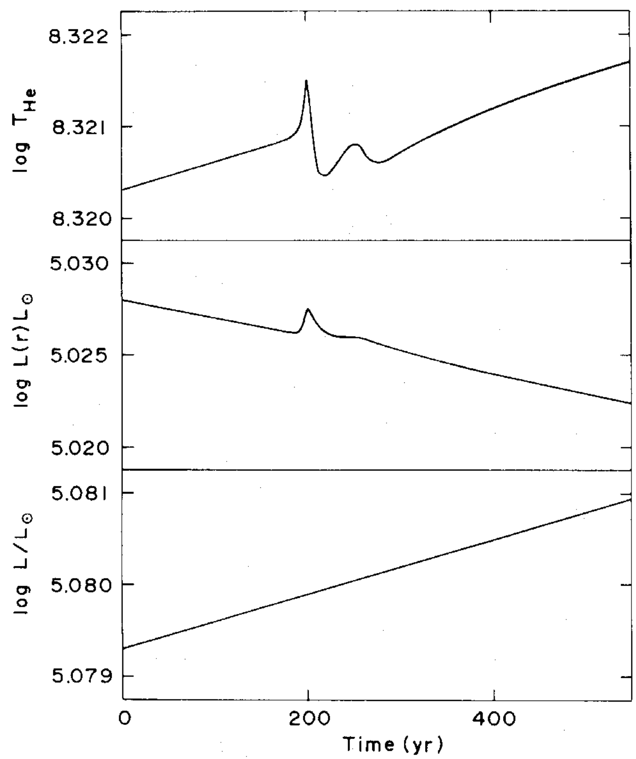

FIG. 2.—Segment of a pulse cycle in the thermally unstable helium-burning shell of a star of 15 $M_\odot$ (sequence 15-O). The panels show, respectively, logarithms of the shell peak temperature, shell peak luminosity, and surface luminosity. This particular cycle happens to exhibit a secondary pulse after the main pulse. Note that the overall evolutionary changes show up here even on the short time scale of a single pulse since the vertical scale of the panels is so expanded.

in the helium-burning shell (the example depicted in fig. 2), and it bears a qualitative resemblance to the secondary pulses found in the helium-burning shell of models for 0.85 $M_\odot$ (Rose and Smith 1970) and 1 $M_\odot$ (Schwarzschild and Härm 1967), though apparently the secondary pulses occur more weakly, or not at all, in the more highly evolved models for 0.85 $M_\odot$ (Rose and Smith 1972) and 1 $M_\odot$ (Sweigart 1971). Usually, the secondary pulses are preceded by a deep "postmaximum" drop of the shell temperature following the main pulse. In our present models, the secondary pulses do not always appear. On the other hand, a rapid succession of pulses (resembling "noise") occasionally does appear. Earlier, Schwarzschild and Härm (1967) had discovered triple pulses in their sequence for 1 $M_\odot$. It is not yet clear what factors govern the determination of the cycle time, the occasional postmaximum drop, and the appearance of secondary or multiple pulses. At least in our sequences, these factors are unrelated to the hydrogen shell, which is extinct throughout most of the phase of carbon-core contraction.

In conclusion, we have shown that thermal instability does temporarily develop in the helium-burning shell of a massive star, shortly after the ignition of shell helium. Manifestation of the instability is an irregular "flickering" of very small amplitude. Its occurrence seems to be independent of uncertainties in the nuclear reaction rates, of neutrino emission, of possible convective overshooting of the helium-burning convective core, and of the presence of the hydrogen shell (which is usually extinct and, at any rate, thermally stable). There are no observable consequences of the instability.

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